AD-A142 185 PROCEDURES FOR SIGNAL RECONSTRUCTION FROM NOISY PHASE (U) GEORGIA INST OF TECH ATLANTA SCHOOL OF ELECTRICAL ENGINEERING D M THOMAS ET AL. 1984 AFOSR-TR-84-0349 1/1

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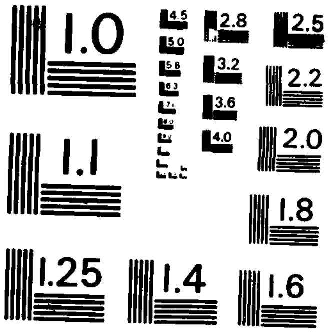

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1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 84-0349
6a. NAME OF PERFORMING ORGANIZATION: Georgia Institute of Technology; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code): School of Electrical Engineering, Atlanta GA 30332	7b. ADDRESS (City, State and ZIP Code): Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: DAAG29-81-K-0024
8c. ADDRESS (City, State and ZIP Code): Bolling AFB DC 20332	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F; PROJECT NO.: 2304; TASK NO.: A9; WORK UNIT NO.

11. TITLE (Include Security Classification)
PROCEDURES FOR SINGAL RECONSTRUCTION FROM NOISY PHASE*

12. PERSONAL AUTHOR(S)
D.M. Thomas and M.H. Hayes

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Technical	FROM ______ TO ______	1984	4

16. SUPPLEMENTARY NOTATION

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17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	pp.1-4 1984

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Joseph Bram	22b. TELEPHONE NUMBER (Include Area Code): (202) 767- 4939; 22c. OFFICE SYMBOL: NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

84 06 18 154

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SECURITY CLASSIFICATION OF THIS PAGE

To be presented at the 1984 International Conference on Acoustics, Speech, and Signal Processing.

AFOSR-TR- 84-0349

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PROCEDURES FOR SIGNAL RECONSTRUCTION FROM NOISY PHASE*

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ABSTRACT

In this paper a class of iterative procedures is presented for reconstructing a finite duration sequence from noisy samples of its Fourier transform phase. These measurements are combined with a knowledge of the true transform magnitude and/or hard constraints on the phase noise variations to define sets whose intersection must contain the true sequence. The algorithms iterate between the sequence domain and the transform domain applying the known constraints (i.e. finite duration and known limits on phase variation) in each domain. Results of an experimental investigation are presented. A method is described for the case where limits on both the magnitude and phase variation of a finite length sequence are known.

INTRODUCTION

In recent years the problem of signal restoration from partial or incomplete information has received considerable attention. In the magnitude retrieval problem, it is assumed that the Fourier transform phase, $\phi(\omega) = \arg[X(\omega)]$, of a finite duration sequence is known exactly at N-1 distinct frequencies in the interval $(0,\pi)$ where N is the known extent of the sequence. Under these conditions and with certain restrictions on the placement of the zeros of $X(z)$, it has been shown [1] that iterative procedures exist which will converge to the unique solution. These algorithms iterate between the sequence domain and the transform domain applying the known constraints (i.e. finite duration and known phase) in each domain. Convergence has been proven within the framework of nonexpansive mapping theory [2].

In contrast, recovering a finite duration sequence $x(n)$ from a knowledge of its Fourier transform magnitude alone has proven to be a much more formidable problem and few sequences meet the uniqueness criterion presently available [3]. This situation has prompted a number of investigators to study the reconstruction of a sequence from samples of its signed magnitude (i.e. the transform magnitude and one bit of phase information) [4]. In this study it has been shown that if $x(n)$ and $y(n)$ are two real, causal (or anticausal), finite duration sequences and if certain restrictions on the placement of the zeros of $X(z)$ are met, then $x(n)$ is equal to $y(n)$ if their respective signed magnitudes are equal at all frequencies. Unfortunately, a given finite set of samples of the signed magnitude is not always sufficient to uniquely specify a sequence. This is true even through it is known that a maximum number of 3N suitably chosen frequency samples suffice to uniquely specify a sequence. The key words are "suitably chosen" since the required samples vary from sequence to sequence. In practice, picking a sufficiently long transform length (typically 10 times the length of the sequence) allows excellent restorations.

When the phase of $X(\omega)$ is not known accurately or when it is corrupted by noise, it has been observed that the magnitude retrieval procedure described above may perform badly. Other techniques, such as reconstruction averaging [5] or minimum cross-entropy methods [6] may also perform poorly, particularly in low signal to noise ratio environments. It seems likely then, that additional information is required if we are to achieve more acceptable performance.

In this paper, a new class of algorithms is presented which achieve significantly better reconstructions over those obtained in the past. These procedures utilize a knowledge of the true transform magnitude and/or bounds on the phase noise variation to define constraint sets whose intersection contains the undistorted signal. At each stage of the iteration, the current estimate of $x(n)$ is projected (in some fashion) onto the constraint sets. In all cases to be examined, the finite duration requirement is enforced in the sequence domain.

Signal Restoration - Noise Free Phase

Let $x(n)$ denote a finite duration one-dimensional sequence which is zero outside the the interval $0 \leq n \leq N-1$. The z-transform and the Fourier transform of $x(n)$ will be denoted by $X(z)$

*This work was supported by the National Science Foundation under Grant ECS-8204793 and by the Joint Services Electronics Program under contract DAAG29-81-K-0024.

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and $X(\omega)$. Since $X(\omega)$ is, in general, a complex-valued function of ω, it may be written in terms of its real and imaginary parts, $X_R(\omega)$ and $X_I(\omega)$ respectively, or in terms of its magnitude and phase, $|X(\omega)|$ and $\phi_x(\omega)$ respectively, as follows:

$$X(\omega) = X_R(\omega) + jX_I(\omega) = |X(\omega)|e^{j\phi_x(\omega)}$$

with: $\quad \phi_x(\omega) = \tan^{-1}\left[X_I(\omega)/X_R(\omega)\right].$

A set of conditions under which $x(n)$ may be recovered from samples of its Fourier transform phase is contained in the following theorem [1]:

> Theorem: Let $x(n)$ be a sequence which is zero outside the interval $0 \leq n \leq N-1$ with $x(0) \neq 0$ and which has a z-transform with no zeros on the unit circle or in conjugate reciprocal pairs. Then samples of the Fourier transform phase $\phi_x(\omega)$ at $N-1$ distinct frequencies in the interval $0<\omega<\pi$ suffice to uniquely (i.e. to within a positive constant) define $x(n)$.

A special case of this theorem arises when the phase samples of $X(\omega)$ are equally spaced around the unit circle such as occurs in the Discrete Fourier Transform (DFT). Let M be the length of the DFT. Then, if $M>2N$, the iterative procedure, which replaces the estimates in each domain by their known values, will converge to the unique solution $x(n)$ for any initial starting point $x_0 \in R^N$ [2]. It is this algorithm that motivates our methods for dealing with noisy phase measurements.

Signal Restoration - Noisy Phase

When noise is added to the phase measurements, the phase-only iterative procedure described above generally produces poor reconstructions. Approaches which use restoration averaging [5] or minimum cross entropy methods [6] also seem to give poor results. This is especially true of these methods in low signal to noise ratio cases. In this section, we will present a number of iterative schemes useful in reconstructing a sequence given noisy phase measurements. These methods have been found to give significantly better reconstructions than have been previously obtained albeit at the cost of higher information requirements. Here the added cost takes the form of knowledge of the true magnitude and/or hard constraints on the phase noise variation. As in the noise-free case, the iteration proceeds by projecting the current estimate of $x(n)$ onto the constraint sets in both the sequence domain and the transform domain. In all cases to be examined, the constraint set in the sequence domain is the knowledge of the first (non-zero) point and the finite duration requirement. The four methods to be described differ only in what constitutes the constraint set in the transform domain.

Given that $\omega_k = 2\pi k/M$, $k = 0,1, \ldots ,M-1$ are a set of M distinct frequencies with $M>2N$, each of the projection operators can be described as follows (see Figure 1 for illustrations of the constraint sets defined below.):

- Method I: No magnitude information is available and the phase noise is known to vary no more then $\pm \Delta_p$ radians from its true value. Alternatively, we can say that the true phase is no more than $\pm \Delta_p$ radians distant from the measured noisy value. At each stage of the iteration, we examine the phase estimate $\phi(\omega_k)$. If it is more than Δ_p radians distant from the measurement value we replace the estimate with the measurement. The magnitude is not altered.

- Method II: Here, the true transform magnitude $|X(\omega_k)|$ is known for all values of k, but, no information is available on the variability of $\phi(\omega_k)$. We do, however, have the noisy phase measurements. For each value of k and at each stage of the iteration we replace the magnitude estimate by its true value. The phase estimate, however, is not changed. It should be pointed out that this procedure differs from the phase retrieval problem, as it usually takes form, in that an initial, if noisy, estimate of the phase is available.

- Method III: Knowledge of the true magnitude is combined in this method with a hard constraint Δ_p on the phase noise variation to define an arc centered on the measured phase value. At each stage of the iteration, the current estimate of $X(\omega_k)$ is projected, using a nearest neighbor rule, onto the convex hull of these arcs.

- Method IV: As in the third method, the true magnitude $|X(\omega_k)|$ is known as is the maximum phase noise variation Δ_p. However, at each stage of the process, the magnitude estimate is replaced by the true value of the magnitude. The phase estimate is replaced by the noisy phase measurement if the estimate lies more than Δ_p radians away from the phase measurement.

To test these four methods, an eight-point non minimum phase sequence $\{x(n) = (4,2,-11,5,4,5,15,-6)\}$ was used. An M-point DFT of this sequence is calculated and to it's phase is added an M-point noise sequence. This noise sequence is generated using a uniform probability density function given by:

$$P_{\Delta_p}(w) = \begin{cases} \frac{1}{2\Delta p}, & -\Delta_p < w < \Delta_p, \\ 0, & \text{otherwise.} \end{cases}$$

Each of the four methods were examined for transform lengths of $M = 16$, 32, 64 and 128 points and noise variations Δ_p equal to $\pi/2$, $10^{-1}\pi$, $10^{-2}\pi$, $10^{-3}\pi$, $10^{-4}\pi$, and $10^{-5}\pi$. A relaxation parameter [2] of 0.99 is used in these experiments. Two performance measures were calculated. They are 1) the normalized mean square error (NMSE) defined by [5]

$$NMSE = \frac{\sum_{n=0}^{N-1} [x(n) - Ax_f(n)]^2}{\sum_{n=0}^{N-1} x^2(n)},$$

and 2) the mean square error improvement ratio (MSEI):

$$MSEI = \frac{\sum_{n=0}^{M-1} [x(n) - x_o(n)]^2}{\sum_{n=0}^{M-1} [x(n) - Ax_f(n)]^2}.$$

In these expressions, $x(n)$ is the true sequence, $x_o(n)$ is the initial or starting estimate of $x(n)$, $x_f(n)$ is the final estimate of $x(n)$, and A is a parameter chosen to minimize the normalized mean square error.

A number of observations can be made from the results obtained thus far in our investigations:

- Methods II and IV perform the best in the noise ranges $10^{-1}\pi$ to $10^{-5}\pi$. For noise variations greater than $10^{-1}\pi$, Method IV is the algorithm of choice. For example, if $\Delta_p = \pi/2$, Method II obtains only 21 dB. of mean square error improvement. Method IV, on the other hand, gives a nearly perfect reconstruction with an MSEI of 242 dB. for M equal to 64 points and 5000 iterations

- At intermediate noise levels (i.e. $10^{-1}\pi$ to $10^{-5}\pi$) methods II and IV give roughly equivalent results. However, as M becomes larger, Method IV tends to converge much more quickly. In the same range of noise levels, Method II and IV converge at the same rate as the phase-only iteration using the sequence without noise added.

- Method III obtains large reductions in the error in the early stages but very little improvement (if any) is obtained as the iteration proceeds.

- Method I is the least effective of the four methods examined. However, it also requires the least amount of a priori information about $x(n)$ or its transform.

- As a rule, increasing M, the length of the transform, improves the convergence rate of the algorithms. However, in some instances, a shorter DFT length can outperform a longer length.

Figure 2 illustrates these observations concerning the four algorithms. Here, M is equal to 64, and for those sequences with noise present it is given a maximum variation of $10^{-3}\pi$ radians. Also shown in this figure is the performance of the phase-only iteration for the phase samples with no phase noise added and with a noise level of $\Delta_p = 10^{-3}\pi$ added. The differences between the initial mean square errors are attributable to whether or not magnitude information is available.

Shown in Figure 3, is the performance of Method IV as Δ_p is varied. Similar effects are seen in the results from the other procedures. In this figure, M is equal to 64.

Signal Restoration - Noisy Phase and Magnitude

In this section, we relax the need for knowledge of the true magnitude and require instead that bounds on the variations in phase and magnitude noise be known. These bounds define a region about the measured magnitude and phase which must contain the true transform values of the sequence. At each stage of the iteration we project, using a nearest neighbor rule, the estimate of $X(\omega_k)$ onto the convex hull of the region described above. Again, in the sequence domain we *enforce* the finite duration requirement. Also, we assume $x(0)$ is known.

Figure 4 represents the results of an experiment using the eight-point sequence defined earlier, Δ_p equal to $10^{-3}\pi$, and M equal to 64. Various levels of uniformly distributed noise ranging from 10^{-1} down to 10^{-5} were added to the magnitude data and reconstructions were obtained as shown. Also shown is the behavior of the phase-only iteration when given the true sequence's phase and when given the true phase corrupted by phase noise at a level of $10^{-3}\pi$.

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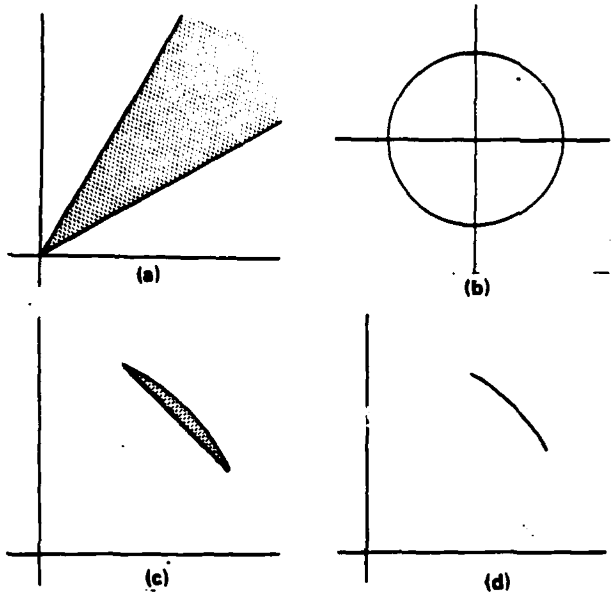

FIGURE 1. CONSTRAINT SETS FOR THE FOUR METHODS DISCUSSED IN THE PAPER: *(a) method I – true sequence* is known to lie in the intersection of M wedge-shaped regions, (b) method II – true sequence lies in the intersection of M circles, (c) method III – true sequence lies in the intersection of the convex hulls of M arcs, and (d) method IV – true sequence is known to lie in the intersection of M arcs.

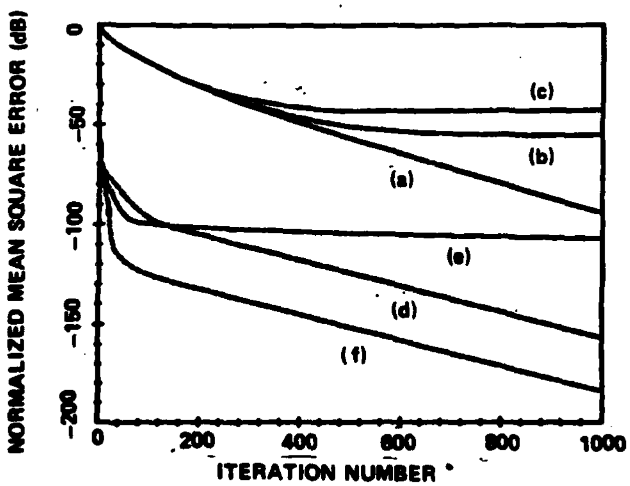

FIGURE 2. COMPARISON OF METHODS (N=8,M=64): (a) phase only iteration – no noise, (b) phase only iteration – noise level of 0.001π, (c) method I – noise level of 0.001π, (d) method II – noise level of 0.001π, (e) method III – noise level of 0.001π, (f) method IV – noise level of 0.001π.

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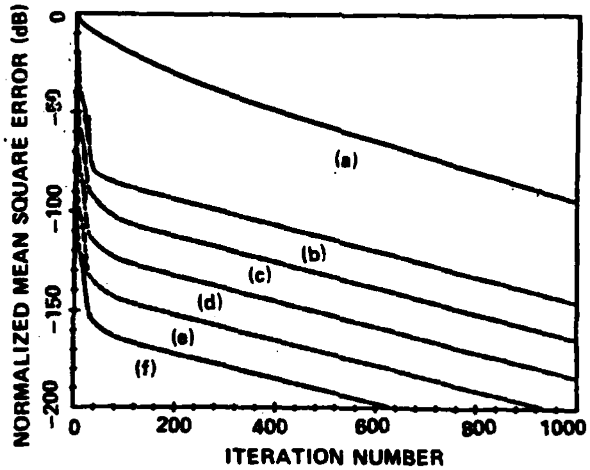

FIGURE 3. EFFECTS OF NOISE LEVEL ON PERFORMANCE USING METHOD IV (N=8,M=64): (a) phase only iteration – no noise, (b) noise level of 0.1π, (c) noise level of 0.01π, (d) noise level of 0.001π, (e) noise level of 0.0001π, (f) noise level of 0.00001π.

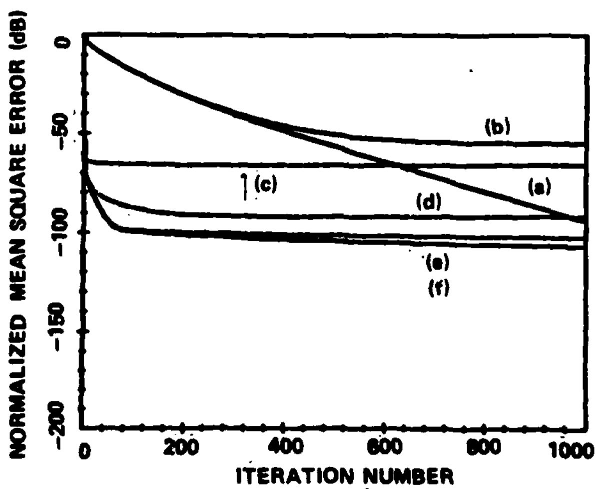

FIGURE 4. RECONSTRUCTION FROM NOISY PHASE AND MAGNITUDE USING CONVEX HULL PROJECTIONS: (a) phase only procedure – no noise, (b) phase only iteration – phase noise level of 0.001π, (c) magnitude noise level of 0.1, (d) magnitude noise level of 0.001, (e) magnitude noise level of 0.00001, and (f) no magnitude noise. (note: unless otherwise specified there is a phase noise level of 0.001π.)